Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Wednesday 13 May 2020

Morning (Time: 1 hour 30 minutes)	Paper Reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have: Mathematical Formulae and Statistical Tables (Lilac), calculator	Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

: explanation
∵ is 'because'
∴ is 'therefore'

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

Pearson

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1. Given that

$$(3pq^2)^4 \times 2p\sqrt{q^8} \equiv ap^bq^c$$

find the values of the constants a, b and c.

(3)

$(3pq^2)^4 \times 2p\sqrt{q^8} = 3^4p^4q^{2\times4} \times 2pq^{\frac{8}{2}}$ ① ②

① indicies rule $a^{bc} = (a^b)^c = (a^c)^b$

② indicies rule $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$\because p^1 = p$

$81p^4q^8 \times 2pq^4 = (81\times2)(p^4\times p^1)(q^8\times q^4) = 162\,p^{4+1}q^{8+4}$ ③ ③

③ indicies rule $a^b \times a^c = a^{b+c}$

$\therefore 162p^5q^{12} = ap^bq^c$

$a = 162$

$b = 5$

$c = 12$

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Question 1 continued

Q1

(Total 3 marks)

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2. $$f(x) = 3 + 12x - 2x^2$$

(a) Express f(x) in the form

$$a - b(x + c)^2$$

where a, b and c are integers to be found.

(3)

The curve with equation $y = f(x) - 7$ crosses the x-axis at the points P and Q and crosses the y-axis at the point R.

(b) Find the area of the triangle PQR, giving your answer in the form $m\sqrt{n}$ where m and n are integers to be found.

(4)

a) Completing the square, if $y = x^2 + bx + c$

it can be rewritten as $y = \left(x + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2$

$f(x) = -2x^2 + 12x + 3$

① factorise $-2\left(x^2 - 6x - \frac{3}{2}\right)$

② Complete the square inside the brackets.

$f(x) = -2\left(\left(x + \left(\frac{-6}{2}\right)\right)^2 + \left(-\frac{3}{2}\right) - \left(-\frac{6}{2}\right)^2\right)$

$= -2\left((x-3)^2 - \frac{3}{2} - (9)\right)$

$= -2\left((x-3)^2 - \frac{21}{2}\right)$

③ expand and write in form $a - b(x+c)^2$

$f(x) = -2(x-3)^2 + (-2)\left(-\frac{21}{2}\right)$

$= -2(x-3)^2 + 21$

$\therefore f(x) = 21 - 2(x-3)^2$ $a = 21$ $b = 2$ $c = -3$

P 6 2 5 9 7 A 0 4 3 2

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Question 2 continued

b) $y = f(x) - 7$ means translation by 7 units down $\begin{pmatrix}0\\-7\end{pmatrix}$. As it is outside $f(x)$ brackets, it only affects y-coordinate values.

$$f(x) - 7 = (21 - 2(x-3)^2) - 7$$

$$\therefore f(x) - 7 = 14 - 2(x-3)^2$$

① find point R. R crosses y-axis $\therefore x = 0$

$f(0) - 7 = 14 - 2((0)-3)^2 = 14 - 2(9) = 14 - 18 = -4$

$R(0, -4)$

② Find points P & Q. Cross x-axis $\therefore y = f(x) - 7 = 0$

$f(x) - 7 = 14 - 2(x-3)^2 = 0$

$$14 - 2(x-3)^2 = 0$$

-14 $\qquad -2(x-3)^2 = -14$ $\qquad -14$

$\div -2$ $\qquad (x-3)^2 = \frac{-14}{-2}$ $\qquad \div -2$

square root $\qquad (x-3)^2 = 7$ $\qquad$ square root

$$x - 3 = \pm\sqrt{7}$$

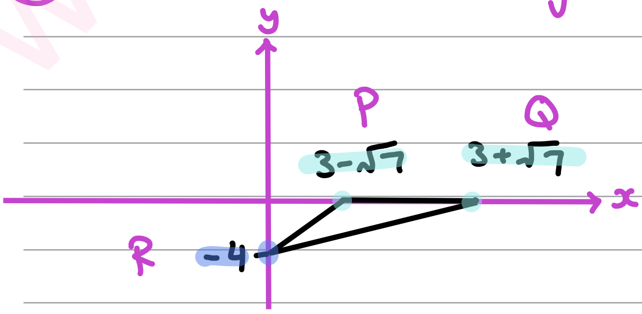
$+3$ $\qquad x = 3 \pm \sqrt{7}$ $\qquad +3$

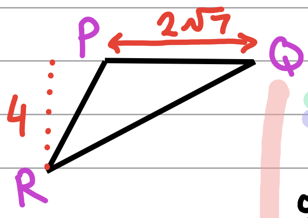
let $x_p = 3 + \sqrt{7}$, $x_q = 3 - \sqrt{7}$

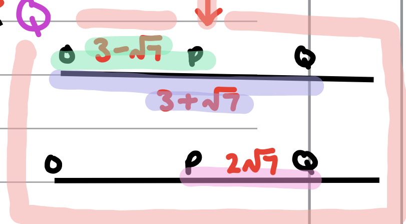
$\therefore P(3+\sqrt{7}, 0) \qquad Q(3-\sqrt{7}, 0)$

③ find area of triangle PQR.

$(3+\sqrt{7}) - (3-\sqrt{7}) = 2\sqrt{7}$







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Question 2 continued

Area of a triangle : $A = \frac{1}{2} bh$

$h = 4$ $\quad A_{PQR} = \frac{1}{2} \times 4 \times 2\sqrt{7} = 4\sqrt{7}$

$b = 2\sqrt{7}$

$\therefore$ Area $= 4\sqrt{7}$ units^2 $\quad m = 4 \quad n = 7$

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Question 2 continued

Q2

(Total 7 marks)

3.

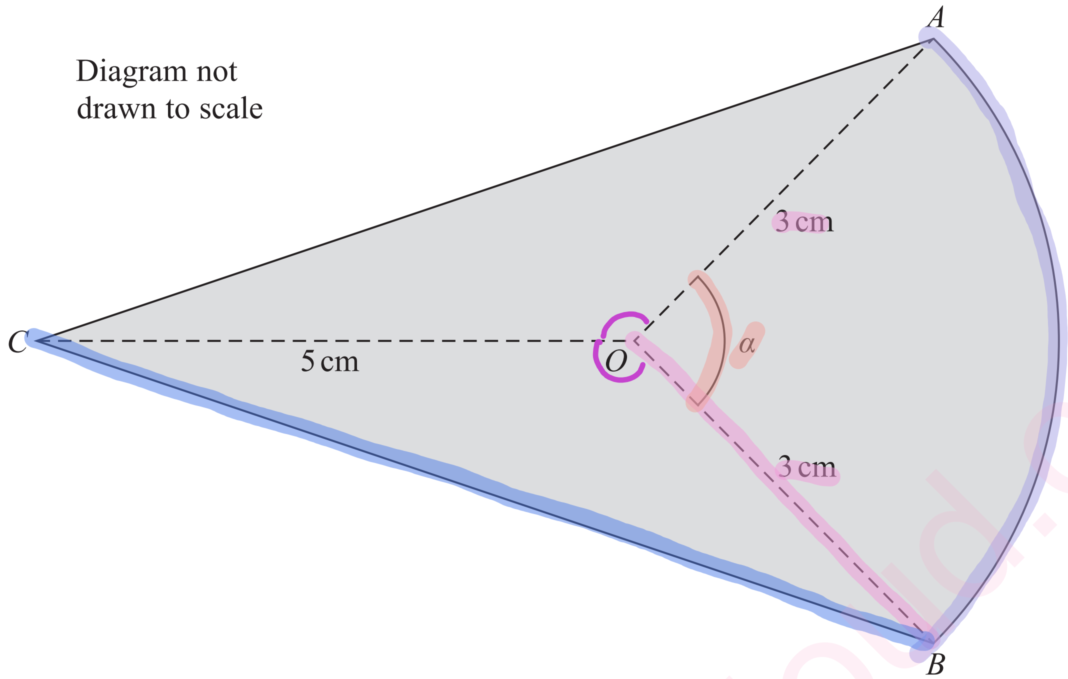


Figure 1

Figure 1 shows the design for a badge.

have the same length sides & angles

The design consists of two congruent triangles, AOC and BOC, joined to a sector AOB of a circle centre O.

- Angle $AOB = \alpha$
- $AO = OB = 3\,\text{cm}$ radius
- $OC = 5\,\text{cm}$

Given that the area of sector AOB is $7.2\,\text{cm}^2$

(a) show that $\alpha = 1.6$ radians.

units

(2)

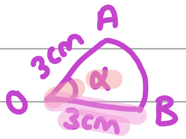
(b) Hence find

(i) the area of the badge, giving your answer in cm^2 to 2 significant figures,

(ii) the perimeter of the badge, giving your answer in cm to one decimal place.

(8)

a) Sector AOB



Area $= 7.2\text{cm}^2$

find $\angle AOB$

Area of sector : $A = \frac{1}{2}r^2\theta$

$7.2 = \frac{1}{2}(3)^2\alpha$

$\div\frac{9}{2}$ $7.2 = \frac{9}{2}\alpha$ $\div\frac{9}{2}$

$1.6 = \alpha$

$\therefore \alpha = 1.6$ rad

P 6 2 5 9 7 A 0 8 3 2

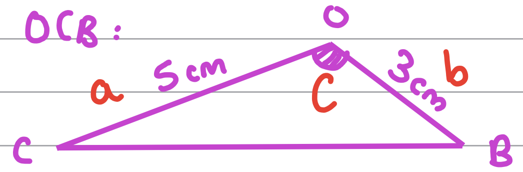
Question 3 continued

b)i) Total Area = $A_{AOB} + A_{OBC} + A_{OAC}$

Area of triangle OCB = OCA $\because$ Congruent triangles

use formula $\frac{1}{2}ab\sin C$ where

triangle OCB:



$C = \angle OCB = \frac{2\pi - \alpha}{2}$

$\angle OCB = \frac{2\pi - 1.6}{2}$

$\therefore A_{OCB} = \frac{1}{2}(5)(3)\sin\left(\frac{\pi - 1.6}{2}\right) = 5.3801\ldots$

Total Area = $A_{AOB} + A_{OBC} + A_{OAC} = 7.2 + 2(5.3801\ldots)$

$= 17.9603\ldots\ \text{cm}^2$

$\therefore$ Total area = $18\ \text{cm}^2$ (2 sig figs)

ii) Perimeter = AC + BC + AB

curved length AB $S = r\theta$

$S_{AB} = 3 \times 1.6 = 4.8\text{cm}$

To find length AC/BC, use cosine rule

from data booklet

Pure Mathematics P1

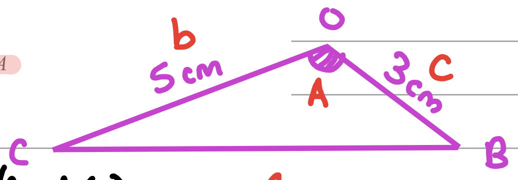
Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$



$a^2 = 5^2 + 3^2 - 2(5)(3)\cos\left(\frac{2\pi - 1.6}{2}\right)$

$BC^2 = 54.9012\ldots$

$BC = \sqrt{54.901\ldots} = 7.4095\ldots$ cm

Question 3 continued

Perimeter = AC + BC + AB $= 2(7.409...) + 4.8$

$= 19.6190...$ cm

$\therefore$ Perimeter = 19.6 cm (1dp)

Leave blank

Question 3 continued

Q3

(Total 10 marks)

Leave blank

4. Use algebra to solve the simultaneous equations

$$y - 3x = 4$$
$$x^2 + y^2 + 6x - 4y = 4$$

You must show all stages of your working.

(7)

① rearrange $y-3x=4$ to make y the subject

$$+3x \quad y - 3x = 4 \quad +3x$$
$$y = 4+3x$$

② Substitute $y=4+3x$ into 2nd equation

$$x^2 + y^2 + 6x - 4y = 4$$

$$x^2 + (4+3x)^2 + 6x - 4(4+3x) = 4$$

③ Simplify

$$x^2 + (9x^2+24x+16) + 6x + (-16-12x) = 4$$
$$x^2 + 9x^2 + 24x + 16 + 6x - 16 - 12x = 4$$

$$10x^2 + 18x = 4$$
$$-4 \quad 10x^2 + 18x - 4 = 0 \quad -4$$
$$2(5x^2+9x-2) = 0$$
$$\div 2 \quad 5x^2 + 9x - 2 = 0 \quad \div 2$$

④ factorise quadratic equation & solve for x.

$$5x^2 + 9x - 2 = 0$$
$$(5x-1)(x+2) = 0$$

$$5x - 1 = 0 \xrightarrow{+1} 5x = 1 \xrightarrow{\div 5} x_1 = \frac{1}{5} = 0.2$$
$$x + 2 = 0 \xrightarrow{-2} x_2 = -2$$

⑤ Substitute x into $y=4+3x$ to find values of y.

$$y_1 = 4 + 3x_1 = 4 + 3\left(\tfrac{1}{5}\right) = \frac{20}{5} + \frac{3}{5} = \frac{23}{5} = 4.6$$

4 0.6

$$y_2 = 4 + 3x_2 = 4 + 3(-2) = 4 - 6 = -2$$

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Question 4 continued

$\therefore \; x_1 = 0.2 \qquad x_2 = -2$

$y_1 = 4.6 \qquad y_2 = -2$

Q4

(Total 7 marks)

5. (i)

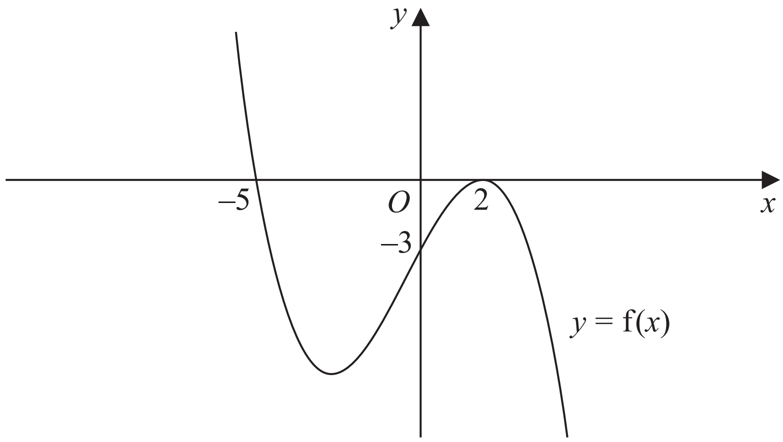


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$.

The curve passes through the points $(-5, 0)$ and $(0, -3)$ and touches the x-axis at the point $(2, 0)$.

On separate diagrams sketch the curve with equation

↙ translation: $\begin{pmatrix}-2\\0\end{pmatrix}$ move left by 2 units

inside f(x) bracket ∴ x coordinate affected

(a) $y = \mathrm{f}(x + 2)$ ∴ $(-7, 0)$ $(-2, -3)$ $(0, 0)$

(b) $y = \mathrm{f}(-x)$ → reflection of original curve in y-ax is

f(x) f(−x)

On each diagram, show clearly the coordinates of all the points where the curve cuts or touches the coordinate axes.

(6)

(ii)

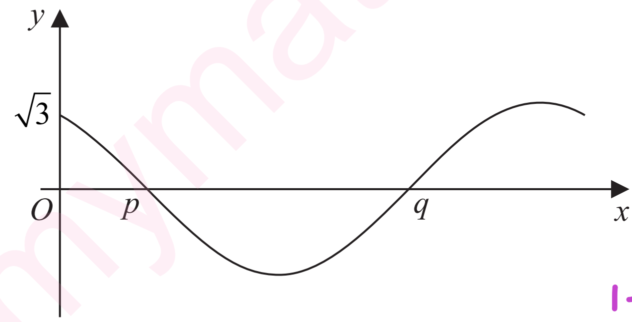


Figure 3

$y = \cos x$

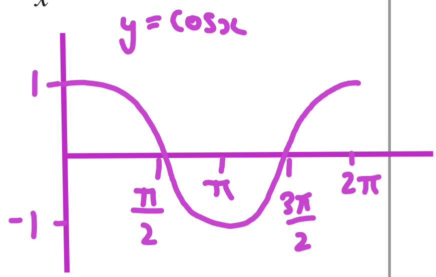


Figure 3 shows a sketch of the curve with equation

$$y = k\cos\left(x + \frac{\pi}{6}\right) \qquad 0 \leqslant x \leqslant 2\pi$$

where k is a constant.

The curve meets the y-axis at the point $(0, \sqrt{3})$ and passes through the points $(p, 0)$ and $(q, 0)$.

Find

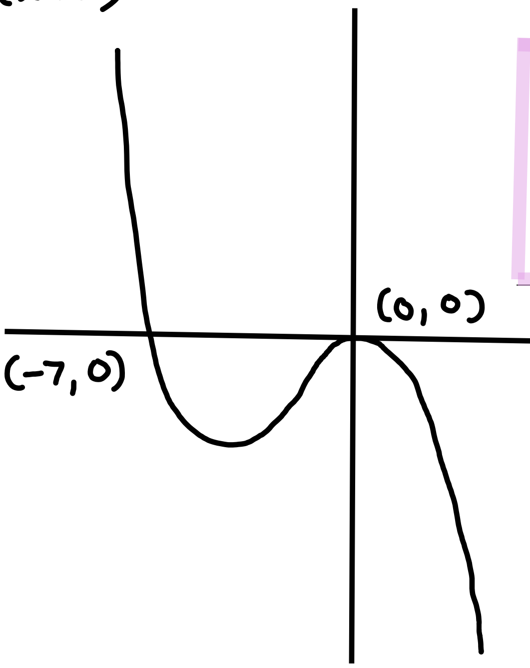
(a) the value of k,

(b) the exact value of p and the exact value of q.

(3)

Question 5 continued

i) a) $y = f(x+2)$



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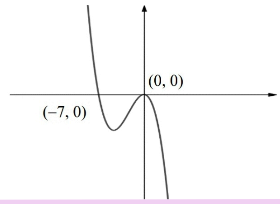


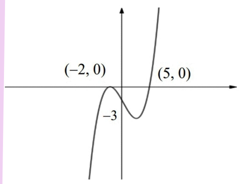
B1 Horizontal translation ←

B1 Maximum at origin

B1 $(-7, 0)$

b) $f(-x)$

Mark scheme:



B1 Reflection in y-axis

B1 Touches at $(-2, 0)$ and passes through $(5, 0)$

B1 Passes through $(0, -3)$

Leave blank

Question 5 continued

Question 5 continued

ii) a) $y = K\cos\left(x + \frac{\pi}{6}\right)$ Maximum is $\sqrt{3}$ when $x=0$

Maximum of $y = \cos x$ is 1 when $x = 0$

let $y = f(x) = \cos x$ $\therefore K f\left(x+\frac{\pi}{6}\right) = K\cos\left(x+\frac{\pi}{6}\right)$

K is outside f(x) brackets $\therefore$ affects y-axis & is vertical stretch.

$K f\left(0 + \frac{\pi}{6}\right) = K\cos\left(0 + \frac{\pi}{6}\right) = K\left(\frac{\sqrt{3}}{2}\right)$

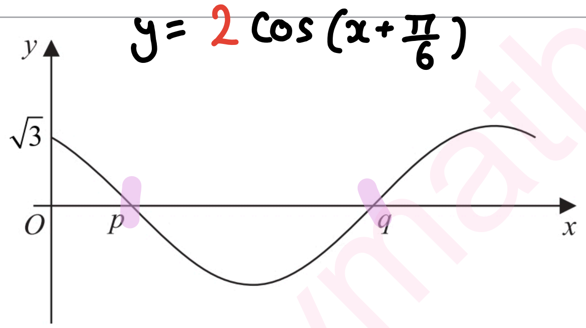
$K\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$ $\quad \div \frac{\sqrt{3}}{2}$

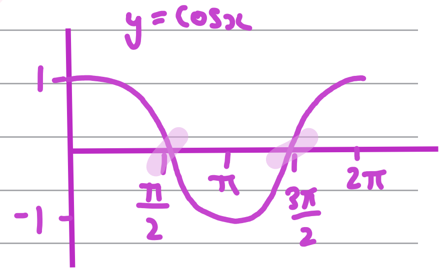
$K = 2$

$\therefore K = 2$

K=2 from part (a)

b) $y = 2\cos\left(x + \frac{\pi}{6}\right)$





P & Q occur when $y = 0$

in $y = \cos x$ graph, this is $\left(\frac{\pi}{2}, 0\right)$ & $\left(\frac{3\pi}{2}, 0\right)$

$y = 2\cos\left(x + \frac{\pi}{6}\right) \Rightarrow y = 2f\left(x + \frac{\pi}{6}\right)$

$\frac{\pi}{6}$ is inside f(x) brackets $\therefore$ affects x-coordinate by translation $\begin{pmatrix} -\frac{\pi}{6} \\ 0 \end{pmatrix}$ move to left by $\frac{\pi}{6}$ units (we do inverse of what is in brackets to x)

$p: \frac{\pi}{2} - \frac{\pi}{6}$ $\qquad q: \frac{3\pi}{2} - \frac{\pi}{6}$

$\therefore p = \frac{\pi}{3}$ & $q = \frac{4\pi}{3}$

Q5

(Total 9 marks)

Turn over

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6. The point A has coordinates $(-4, 11)$ and the point B has coordinates $(8, 2)$.

(a) Find the gradient of the line AB, giving your answer as a fully simplified fraction.
(2)

The point M is the midpoint of AB. The line l passes through M and is perpendicular to AB.

(b) Find an equation for l, giving your answer in the form $px + qy + r = 0$ where p, q and r are integers to be found.

↳ answer in whole numbers
(4)

The point C lies on l such that the area of triangle ABC is 37.5 square units.

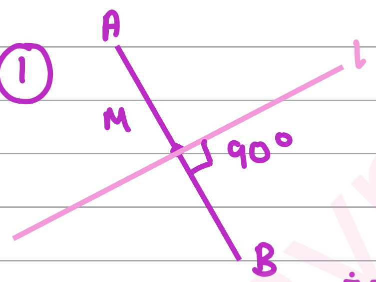
(c) Find the two possible pairs of coordinates of point C.
(5)

a) gradient formula $m = \dfrac{y_1 - y_2}{x_1 - x_2}$

$m_{AB} = \dfrac{11 - 2}{-4 - 8} = \dfrac{9}{-12} = -\dfrac{\cancel{3}(3)}{\cancel{3}(4)} = -\dfrac{3}{4}$

$\therefore$ gradient $AB = -\dfrac{3}{4}$

b) 1



l is normal to AB

gradient of normal m_n:

$m_n \times m = -1$

$\div -\frac{3}{4}$ $m_n \times \left(-\frac{3}{4}\right) = -1$ $\div -\frac{3}{4}$

$m_n = \dfrac{4}{3}$

2 Find point M, midpoint: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 - y_2}{2}\right)$

$M: \left(\dfrac{-4 + 8}{2}, \dfrac{11 + 2}{2}\right) = \left(\dfrac{4}{2}, \dfrac{13}{2}\right)$

$\therefore M\left(2, \dfrac{13}{2}\right)$

3 Find equation of l using line passing through (a, b) and gradient m

equation: $(y - b) = m(x - a)$

Question 6 continued

$a = 2$
$b = 13/2$
$M = 4/3$

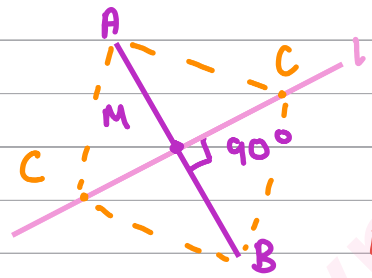
$$\left(y - \frac{13}{2}\right) = \frac{4}{3}\left(x - 2\right)$$

(4) write in the form $px + qy + r = 0$

$$\begin{aligned}
\times 3 \quad y - 13/2 &= 4/3\,(x-2) \quad \times 3\\
\times 2 \quad 3y - 39/2 &= 4\,(x-2) \quad \times 2\\
6y - 39 &= 8\,(x-2)\\
-6y \quad 6y - 39 &= 8x - 16 \quad -6y\\
+39 \quad -39 &= 8x - 6y - 16 \quad +39\\
0 &= 8x - 6y + 23
\end{aligned}$$

$\therefore 8x - 6y + 23 = 0$ $\quad p = 8 \quad q = -6 \quad r = 23$

c)



2 possible locations of C

Area is 37.5 units^2

Area of a triangle : $A = \frac{1}{2}bh$

distance between 2 points : $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$b = |\vec{AB}| = \sqrt{(-4-8)^2 + (11-2)^2} = \sqrt{(-12)^2 + (9)^2} = \sqrt{144+81}$
$= \sqrt{225} = 15$

$h = |\vec{MC}|$

$$A = \frac{1}{2}bh = \frac{1}{2} \times \vec{AB} \times \vec{MC}$$

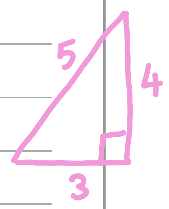
$$\tfrac{1}{2} \times 15 \times \vec{MC} = 37.5$$
$$\tfrac{15}{2} \times \vec{MC} = 37.5$$
$$\div \tfrac{15}{2} \quad \vec{MC} = 5 \quad \div \tfrac{15}{2}$$

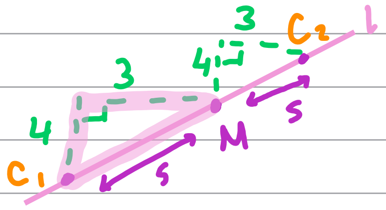
Leave blank

Question 6 continued

$h = |\vec{MC}| = \sqrt{(x-2)^2 + \left(y - \frac{13}{2}\right)^2} = 5$

$M_n = \frac{4}{3}$ $\quad \vec{MC} = 5 \rightarrow 4^2 + 3^2 = 5^2$





gradient is rise (↑) over run (→)

$M\left(2, \frac{13}{2}\right)$

$C_1 : \left(2-3, \frac{13}{2} - 4\right) = \left(-1, \frac{5}{2}\right) = (-1, 2.5)$

$C_2 : \left(2+3, \frac{13}{2} + 4\right) = \left(5, \frac{21}{2}\right) = (5, 10.5)$

$\therefore C(-1, 2.5)$ or $C(5, 10.5)$

Leave blank

Question 6 continued

Q6

(Total 11 marks)

7. The curve C has equation

$$y = \frac{1}{2 - x}$$

(a) Sketch the graph of C. On your sketch you should show the coordinates of any points of intersection with the coordinate axes and state clearly the equations of any asymptotes.

(3)

The line l has equation $y = 4x + k$, where k is a constant.

Given that l meets C at two distinct points,

(b) show that

$$k^2 + 16k + 48 > 0$$

(4)

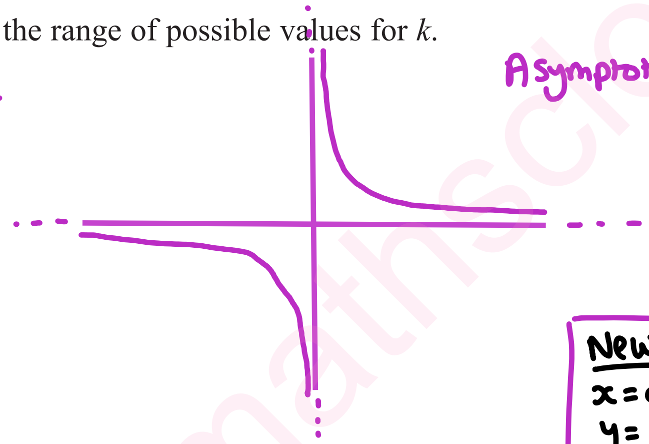
(c) Hence find the range of possible values for k.

(4)

when $y = \frac{1}{x}$

Asymptotes : $y = 0$

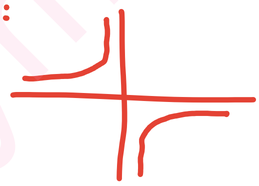
$x = 0$



$y = \frac{1}{2-x}$

negative reciprocal :

translate 2 units to right $\binom{2}{0}$



New asymptotes :

$x = 0 + 2 = 2$

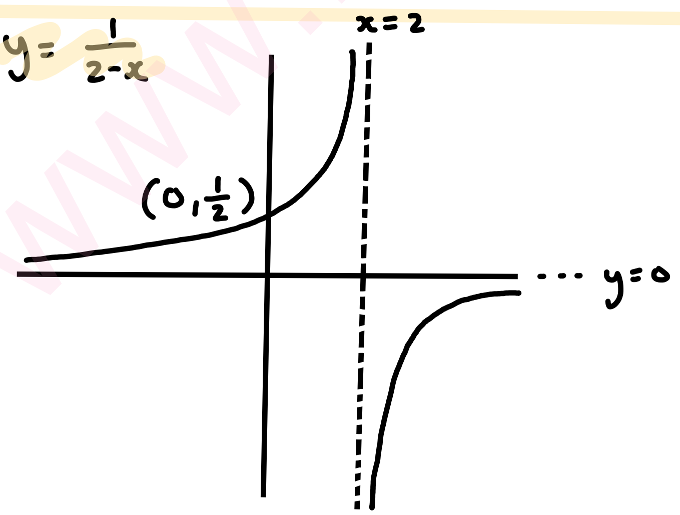
$y = 0$

Any intercepts :

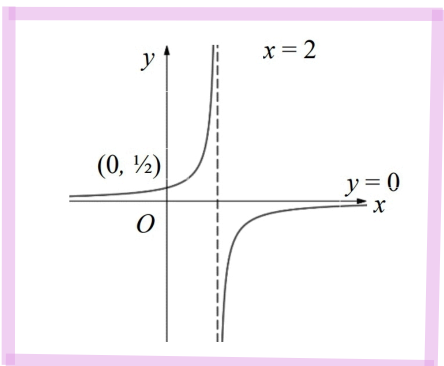
when $x = 0$, $y = \frac{1}{2}$

when $y = 0$, x undefined

$y = \frac{1}{2-x}$



Mark scheme :



P 6 2 5 9 7 A 0 2 2 3 2

Question 7 continued

b) l meets C 2 times $\therefore$ discriminant is

$b^2 - 4ac > 0$ for 2 real roots

① equate both equations & simplify.

$$\times(2-x)\quad 4x + k = \frac{1}{2-x} \quad \times(2-x)$$

$$(4x+k)(2-x) = 1$$

$$-1 \quad (4x+k)(2-x) - 1 = 0 \quad -1$$

$$(8x - 4x^2 + 2k - kx) - 1 = 0$$

$$-4x^2 + 8x - kx + 2k - 1 = 0$$

② Use discriminant formula

$$-4x^2 + (8-k)x + (2k-1) = 0$$

$a = -4$

$b = (8-k)$

$c = (2k-1)$

$$b^2 - 4ac > 0$$

$$(8-k)^2 - 4(-4)(2k-1) > 0$$

$$(k^2 - 16k + 64) - 4(-8k + 4) > 0$$

$$(k^2 - 16k + 64) + 32k - 16 > 0$$

$$k^2 + 16k + 48 > 0$$

$$\therefore k^2 + 16k + 48 > 0$$

c) $k^2 + 16k + 48 > 0$

factorise : $(k+12)(k+4) > 0$

find critical values :

$k + 12 = 0 \longrightarrow k = -12$ $\quad k \neq -12 \quad \because (k+12)(k+4) > 0$

$k + 4 = 0 \longrightarrow k = -4$ $\quad k \neq -4 \quad$ > not ≥ !!

when k is 1 less than $-12 \Rightarrow k = -13$

$(-13+12)(-13+4) = 9 \quad 9 > 0$ TRUE $\quad \therefore k < -12$ NOT ≤

Leave blank

Question 7 continued

when K is 1 less than −4 ⟹ $K = -5$

$(-5+12)(-5+4) = -7$ $-7 > 0$

FALSE So K CANNOT be less than −4

NOT ≥

$\therefore \; K > -4$

$\therefore \; K < -12$ & $K > -4$

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Question 7 continued

Q7

(Total 11 marks)

Turn over

8. The curve C has equation

$$y = (x - 2)(x - 4)^2$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 20x + 32$$

(4)

The line l_1 is the tangent to C at the point where $x = 6$

(b) Find the equation of l_1, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

The line l_2 is the tangent to C at the point where $x = \alpha$

Given that l_1 and l_2 are parallel and distinct,

(c) find the value of α

(3)

a) ① Expand brackets

$y = (x-2)(x-4)^2 = (x-2)(x^2-8x+16) = x^3 - 8x^2 + 16x - 2x^2 + 16x - 32$

$= x^3 - 10x^2 + 32x - 32$

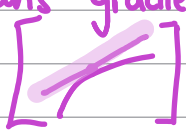
② differentiate

$dy/dx = 3(x^{3-1}) + 2(-10x^{2-1}) + 1(32x^{1-1}) + 0(-32x^{0-1})$

$= 3x^2 - 20x + 32$

$\therefore dy/dx = 3x^2 - 20x + 32$

b) tangent means gradient of tangent is same as gradient of equation



l_1 intersects with C at $x = 6$. Substitute $x = 6$ with equation C.

$y = (x-2)(x-4)^2 = (6-2)(6-4)^2 = (4)(2)^2$

$= (4)(4) = 16$

$\therefore l_1$ intersects C at $(6, 16)$

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Question 8 continued

① to find gradient of tangent, Substitute x-value of l_1 into dy/dx (The gradient function)

from part (a)

$$\left.\frac{dy}{dx}\right|_{x=6} = 3(6)^2 - 20(6) + 32 = 20$$

② find equation of tangent using line passing through (a, b) and gradient m

equation: $(y - b) = m(x - a)$

$a = 6$
$b = 16$
$m = 20$

$$(y - 16) = 20(x - 6)$$

③ write in the form $y = mx + c$

$$y - 16 = 20(x - 6)$$
$$y - 16 = 20x - 120$$
$$+16 \quad y = 20x - 104 \quad +16$$

$\therefore y = 20x - 104$

c) $l_1 \parallel l_2$ (parallel) $\therefore$ have same gradient.

$\therefore$ gradient of l_2 is 20

hence, when dy/dx (gradient function) $= 20$, $x = \alpha$

Same dy/dx of C $\because$ l_2 is tangent to C

$$\therefore \left.\frac{dy}{dx}\right|_{x=\alpha} = 3\alpha^2 - 20\alpha + 32 = 20$$

Question 8 continued

$$-20 \quad 3\alpha^2 - 20\alpha + 32 = 20 \quad -20$$
$$3\alpha^2 - 20\alpha + 12 = 0$$

Factorise : $(3\alpha - 2)(\alpha - 6) = 0$

Solve : $3\alpha_1 - 2 = 0$

$+2 \quad 3\alpha_1 = 2 \quad +2$

$\div 3 \quad \alpha_1 = \frac{2}{3} \quad \div 3$

$\alpha_2 - 6 = 0$

$\boxed{\alpha_2 = 6}$

$x = 6$ is where l_1 is tangent to C.

$\therefore \alpha = \frac{2}{3}$

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Question 8 continued

(Total 11 marks)

Q8

P 6 2 5 9 7 A 0 2 9 3 2

Turn over

9. A curve with equation $y = \mathrm{f}(x)$ passes through the point (9, 10).

Given that

$$\mathrm{f}'(x) = 27x^2 - \frac{21x^3 - 5x}{2\sqrt{x}} \qquad x > 0$$

find $\mathrm{f}(x)$, fully simplifying each term.

(6)

$$\mathrm{f}'(x) \xrightarrow{\text{integrate}} \mathrm{f}(x), \qquad \mathrm{f}(x) \xrightarrow{\text{differentiate}} \mathrm{f}'(x)$$

$$\therefore \int \mathrm{f}'(x)\,dx = \mathrm{f}(x)$$

① write f'(x) in easier form to integrate.

$$\mathrm{f}'(x) = 27x^2 - \left(\frac{21x^3 - 5x}{2\sqrt{x}}\right) = 27x^2 - \frac{21x^3 + 5x}{2x^{\frac{1}{2}}}$$

① indicies rule : $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$$\mathrm{f}'(x) = 27x^2 - \left(\frac{1}{2} \times \frac{-21x^3 + 5x}{x^{1/2}}\right)$$

$$= 27x^2 - \left(\frac{1}{2} \times \left(-21x^{3-\frac{1}{2}} + 5x^{1-1/2}\right)\right)$$

② indicies rule : $\frac{a^b}{a^c} = a^{b-c}$

$$\mathrm{f}'(x) = 27x^2 + \frac{1}{2}\left(-21x^{\frac{5}{2}} + 5x^{1/2}\right)$$

$$= 27x^2 - \frac{21}{2}x^{\frac{5}{2}} + \frac{5}{2}x^{1/2}$$

② integrate

$$\int 27x^2 - \frac{21}{2}x^{5/2} + \frac{5}{2}x^{-1/2}\,dx = \left[\left(\frac{27}{2+1}x^{2+1}\right) - \left(\frac{(21/2)}{5/2+1}x^{\frac{5}{2}+1}\right) + \left(\frac{(5/2)}{\frac{1}{2}+1}x^{\frac{1}{2}+1}\right)\right]$$

$$= 9x^3 - 3x^{\frac{7}{2}} + \frac{5}{3}x^{\frac{3}{2}} + C$$

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Question 9 continued

③ find value of constant C by substituting $x = 9$ & $y = f(x) = 10$ from point $(9, 10)$

$$f(9) = 9(9)^3 - 3(9)^{\frac{7}{2}} + \frac{5}{3}(9)^{\frac{3}{2}} + C = 10$$

$$-45 \quad 45 + C = 10 \quad -45$$

$$C = -35$$

$$\therefore f(x) = 9x^3 - 3x^{\frac{7}{2}} + \frac{5}{3}x^{\frac{3}{2}} - 35$$

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Question 9 continued

Q9

(Total 6 marks)

TOTAL FOR PAPER IS 75 MARKS

END